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Lexicographic Product of Standard Graphs Using Claw Decompositions

[1] C.Sujatha and [2] A.Manickam
[1] *Principal, Professor of Mathematics, Marudu Pandiyar College, Vallam Post, Thanjavur–613 403, Tamilnadu, India.*
Email id: sujatha2276@gmail.com
[2] *Assistant Prof of Mathematics, Marudu Pandiyar College, Vallam Post, Thanjavur–613 403, Tamilnadu, India.*
Email id: manickammaths2011@gmail.com

Abstract- In this paper, we shall discuss necessary and sufficient condition for the decomposition of Cartesian products of standard graphs into claws. Also, we give a sufficient condition for the claw decomposition of Lexicographic product of standard graphs.

Keywords- Claw decomposition, Cartesian product, Lexicographic product.

1. INTRODUCTION

Let $G = \{V, E\}$ be a simple undirected graph without loops or multiple edges. A path on n vertices is denoted by p_n,cycle on n-vertices is denoted by c_n and complete graph on n vertices is denote by k_n.A decomposition of a graph G is a family of Edge–disjoint subgraphs $\{G_{1,} G_{2,} G_{3,} G_{4,} \ldots\ldots G_{k,}\}$ such that $E(G) = E(G_1) U\ E(G_2) \ldots\ldots U\ E(G_k)$.If each G_i is isomorphic to H for some subgraph H of G,then the decomposition is called a H-decomposition of G. If H has at least three edges, then the problem of deciding if a Graph G has a H-decomposition is NP-Complete[2].A star with three edges is a called a claw. A complete bipartite graph with partite sets V_1 and V_2,where $|V_1| = r$ and $|v_2| = S$,is denoted by $K_{r,s}$.The Graph $K_{1,r}$ is called star and is denoted by S_r.The vertex of degree r in the star S_r is called the central vertex of the star.The complement of a graph G is denoted by $\overleftarrow{G}.\ K_G$ denotes the union of k copies of G.The join $G + H$ of two graphs G and H consists of $G\ U\ H$ and all edges joining each vertex of G to all the vertices of H.Terms not defined here are used in the sense of [4].

If h has at least three edges,then the problem of deciding if a Graph G has a H-decomposition is NP-Complete[1,2]..

In 1975, Sumiyasu, yamamoto etal.., gave necessary and sufficient condition for the S_k-decomposition of complete graphs and complete bipartite graphs. In 1996, C.Lin and T.W Shyu[4]presented a necessary and sufficient condition for decompositions K_n into stars

$S_{k_1},\ S_{k_2} \ldots\ldots.\ S_{k_t}$

In 2004,H.L.Fu et al .,[3] decomposed a complete graph into Cartesian product of two complex graphs $K_{r,}K_t$.

In 2012,Darry E.Bryant et al[1] gave necessary and sufficient condition for the existence of K-star factorizations of any power ${K_q}^s$,where q is prime and the products

$C_{r_1} X\ C_{r_2} X\ \ C_{r_3} \ldots.\ C_{r_k}$of k cycles of arbitrary length.

In 2013 Tay-Woei shyu[5] gave necessary and sufficient condition for the decomposition of complete graph into C_1’s and S_k’s.

In this paper, we give necessary and sufficient condition for decomposition of Cartesian product of standard graphs into claws. Also we give a sufficient condition for the claw-decomposition of lexicographic product of standard Graphs.

2. PRELIMINARIES

In this section,We collect ceratin lemma’s and results which are used in the subsequent sections. These are the building blocks in the construction of the main theorems.

Definition 2.1

The corona of two graphs G and H,is the graph GoH formed from one copy of G and |V(G)| copies of H where the i^{th} vertex of G is adjacent to every vertex in the i^{th} copy of H.

Definition 2.2

The Cartesian product of two Graphs g and H is a graph, denoted by

G x H, whose vertex set is V(G)xV(H).Two vertices (g.h) and (g^1,h^1) are adjacent precisely if $g=g^1$ and $h\ddot{h} \in E(H)$ or $g\ddot{g} \in E(G)$ and $h=h^1$.

Thus V(GxH)={(g.h)/g $\in$ V(G) and h $\in$ V(H)}

E(VxH)={(g,h)(g^1,h^1)/$g=g^1$ and $hh^1 \in$ E(H) or $gg^1 \in E(G) and\ h = h^1$}

Theorem 2.3

A complete Graph ,k_1 with l pairs and$(l/2)$ lines can be decomposed into a union of line disjoint$(l/2)/c$ claws $K_{1,c}$ with l lines each⟺

(i)(l/c) is an integral multiple of c and
(ii) $l \geq 2c$.

Theorem 2.4

A complete bigraph $K_{m,n}$ with m and n points and mn lines each the decomposed into union of mn/c line disjoint $(l/2)/c$ claws k_1,c with c lines each ⟺if m and n satisfy one of the following three conditions.
(i)n≡0(mod c)when m<c
(ii)m≡0(mod c)when n<c
(iii)mn≡0(mod c)when m ≥ c and n ≥ c

Theorem 2.5

The Graph C_n o $\overline{k_2}$ is claw decomposable for all n.

Proof:

Let $V(C_n)=\{V_1,V_2,\ldots.V_n\}$And let u_i and w_i be the pendant vertices at V_i. then $<\{u_{i,}w_{i,}v_{i,}v_{i+1,}\} \cong k_{1,3}$ for all $1\leq i \leq n-1$ and $<\{u_{n,}w_{n,}v_{n,}v_{1,}\} \cong k_{1,3}$
Thus E(Cn 0 $\overline{k_2}$)=E($k_{1,3}$) U …U E($K_{1,3}$).
Hence Cn 0 $\overline{k_2}$ is claw decomposable

Theorem 2.6

If n is even and $n \equiv 0\ (mod\ 3)$ then k_2 x C_n is claw decomposable.

Proof:

Let $V(K_2)=\{x_1,x_2\}$
And let $V(C_n)=\{y_1,y_2\ldots.y_n\}$ then $V(k_2$ X $V_n)=\{(x_{i,}y_j)/i=1,2$ and $1\leq j\leq n$
Rename $(x_1,y_j)=v_j$ and $(x_2,y_j)=u_j$ ∀ 1≤j≤n..
Now
$<\{v_1,v_2,v_n,u_1\}>\cong K_{1,3}$
$<\{u_1,u_{n-1},u_n,v_n\}>\cong K_{1,3}$
$<\{u_{i+1},v_i,v_{i+1},v_{i+2}\}>\cong K_{1,3} \forall$ i ∈{2,4,6…n-2}
And
$<\{ u_i,u_{i+1}, u_{i+2,} u_{i+1} \}>\cong \forall$ i∈{1,3,5,7….n-3}
Thus E(k_2 X C_n)=E($k_{1,3}$) U….U E(k_1)
Hence k_2 X C_n is claw decomposable.

3. CLAW DECOMPOSITION OF CARTESIAN PRODUCT OF GRAPHS

In this section we give necessary and sufficient condition for the decomposition of Cartesian product of some standard graphs into claws.

Theorem 3.1

If G_1 and G_2 are H decomposable then G_1 X G_2 is H decomposable.

Proof:

Let $V(G_1)=\{V_1,V_{2,\ldots.}V_K\}$ and $V(G_2)=\{u_1,u_2\ldots.u_n)$ then $V(G_1\ XG_2)=\{(v_{i,}u_j)/1\leq i\leq k,1\leq j\leq n\}$
Rename $(v_i,u_j)=v_{ij}$; 1≤i≤k,1≤j≤n
Now
$<\{v_{1,j}\ ,v_{2,j}\ ,\ldots v_{k,j}\}>\cong G_1 \forall\ 1\leq j\leq n$
$<\{u_{i,1}\ ,u_{i,2}\ ,\ldots u_{i,n}\}>\cong G_2 \forall\ 1\leq i\leq k$
Thus E($G_{1\ X}G_2$)=E(G_1) U …..U E(G_1) U E(G_2) U….U E(G_2)
Since G_1 and G_2 is H decomposable

Corollary 3.2

If m,n $\equiv 0(mod\ 3)$ then $k_{1,m}$ X $k_{1,n}$ is claw decomposable.

Corollary 3.3

If m $\equiv 0(mod\ 3) and$ n≠ 2 (mod 3) then $k_{1,m}$ X k_n is claw decomposable.

Proof:

It follows the theorem 2.3.and 3.1

Corollary 3.4

If rs $\equiv 0(mod\ 3) and$ claw n≡2 (mod 3) then $k_{r,s}$ X k_n is claw decomposable.

Proof:

It follows the theorem 2.3,2.4and 3.1

Corollary 3.5

If rs $\equiv 0(mod\ 3) and$ claw n≡0 (mod 3) then $k_{r,s}$ X $k_{1,n}$ is $k_{1,3}$ decomposable.

Proof:

It follows from theorem 2.4 and 3.1

4. CLAW DECOMPOSITION OF LEXICOGRAPHIC PRODUCT OF GRAPHS

In this section, we give sufficient condition for the lexicographic product of any Graphs G with $\overline{k_n}$,k_n,$k_{m,n}$ and k_2 x k_n to be claw decomposable.

Definition 4.1

The lexicographic product of two Graphs G and H is a Graph denoted by G * H,whose vertex set is V(G) x V(H).Two vertices (g,h) and $(\dot{g},\dot{h})$ are adjacent precisely if $g\dot{g}$ ∈ E(G),or g=$\dot{g}$ and h$\dot{h}$ ∈ E(H).

The other way of viewing G * H is by replacing each vertex in G by a copy of H and two vertices in G are adjacent if and only if if there exists a complete bipartite sub graph with the corresponding vertices of H as partite sets in G*H.

Theorem 4.2

Let G be any non-trivial Graph.It n≡ $0(mod\ 3)$,then G* $\overline{k_n}$ is claw decomposable.

Proof:

Assume that n≡ $0(mod\ 3)$
Let $V(G)=\{v_{1,}v_{2\ldots.}v_k\}$
And $V(\overline{k_n}) = \{u_1,u_{2\ldots.}u_n\}$
Then $V(G*\overline{k_n}) = \{ (v_{i,}u_j)/\ 1\leq i\leq k,1\leq j\leq n \}$
Rename
$(V_{i,}u_j)=V_{ij}$; 1≤i≤k,1≤j≤n
Now for each $v_{i,}v_j$ ∈ E(G)$<\{v_{1,i,}v_{2,i\ldots..}v_{n.i,}v_{1,j,}v_{2,j\ldots..}v_{n,j}\}>\cong k_{n,n}$
Thus E(G*$\overline{k_n}$)=E($k_{n,n}$)U….UE($k_{n,n}$)
Since n≡ $0(mod\ 3), by\ thorem\ 2.4\ K_{n,n}$ is claw decomposable.
Hence G*$\overline{k_n}$ is claw decomposable

Theorem 4.3

Let G be any non-trival graph.If n>3 and n≡ $0(mod\ 3)$,then G*K_n is claw decomposable.

Proof:

Assume that n>3 and $n \equiv 0 \bmod 3$
Let $V(G)=\{v_{1,}v_{2,\dots}v_k)$
And $V(K_n)= \{u_{1,}u_{2,\dots}u_n)$
Then $V(G^* K_n)= (v_{i,}u_j)/\ 1\leq i\leq k, 1\leq j\leq n\ \}$
Rename
$(V_{i,}u_j)=V_{ij};\ 1\leq i\leq k, 1\leq j\leq n$
Now
$<\{\ v_{1,}v_{2,\dots}v_n\}>\cong k_n \forall 1\leq i\leq k,$
Also for each $v_{i,}v_j \in E(G)$
$<\{v_{1i,}\ v_{2i,}\ \dots v_{ni}\ ,v_{1j,}\ v_{2j\dots}\ v_{ng}>$
$E(<\{V_{1i},V_{2i\dots\dots}v_{ni}>)$
$E(<\{V_{1j},V_{2j\dots\dots}v_{nj}>)\cong k_{n,n}$
Thus $E(G * K_n)=E(K_n)\ U\dots U\ E(K_n)\ U\ E(K_{n,n})$
Since $n\equiv 0\ mod(3) by\ them\ 2.3\ and\ 2.4$ and $k_{n,n}$ are claw decomposable .
Hence $G * K_n$ is claw decomposable.

5. CONCLUSION

In this paper, we shall discuss about claw decomposition of Product graphs. We give necessary and sufficient condition for the decomposition of Cartesian product stdand ard and ordinary graphs into claws. Also we give a sufficient condition for the claw decomposition of lexicographic product of standard and ordinary graphs has been clearly understood and discussed in this research article.

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